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SOME PROBLEMS OF ULTRA-SONIC WAVES

BY

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A RESEARCH CARRIED OUT UNDER THE

DIRECTION OF

DEAN R. W. BOYLE.

Physics Laboratory,

University of Alberta,

Edmonton,

April, 1923.

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SOME PROBLEMS OF ULTRA-SONIC WAVES.

INTRODUCTION.

Ultra-sonic waves are longitudinal waves of very high pitch, well above the limits of audibility. Such waves when set up in a fluid have very short wave-lengths ranging from a fraction of a centimeter to five or more centimeters and therefore can be reflected, refracted, and diffracted, similarly to light waves.

When a disk or diaphragm executes high frequency vibrations in a medium, if all the points of the face of the diaphragm vibrate in the same phase and the diameter of the diaphragm is a number of times the wave-length, then most of the ultra-sonic energy is radiated into the medium from the face of the diaphragm in the form of a beam, whose angular width is determined from the relation $\sin \theta = \frac{1.2}{D} \lambda$, where θ is half the angle of the beam, λ is the wave-length of the supersonic wave and D is the diameter of the diaphragm.

It can be shown mathematically that when ultra-sonic waves in a fluid strike an object which has a density and elasticity much different from those of the fluid, the waves suffer reflection, and if the reflecting object is a plane surface at right angles to the beam and correctly positioned, the advancing and reflected waves meet and interfere to form standing waves.

The wave-lengths of ultra-sonic waves are intermediary to the long wave-lengths of sound and the extremely short wave-lengths of light. Using an appropriate type of oscillator these waves can very conveniently be used for investigating the variation in the velocity of the wave with change of medium, the compressibility of the medium, its viscosity, and other related problems.

Generation of Ultra-Sonic Waves.

If a parallel beam of light is passed normally through a small circular opening in an opaque screen, alternate zones of reinforcement and neutralization are formed due to interference. The position of these zones of maximum and minimum intensities depends upon the diameter of the opening and the wave-length of the energy passing through it. If ordinary sound waves pass through the hole, interference does not take place but the energy is distributed

uniformly in all directions, that is, the hole becomes the center of the emergent waves. This latter phenomenon is due to the relative size of the hole and the wave-lengths involved. If the wave-length is much greater than the radius of the hole the energy is distributed spherically, as in the case of sound waves of ordinary pitch passing through an aperture. But if the radius is large compared to the wave-length, diffraction takes place. Therefore if sound waves can be obtained with a sufficiently small wave-length through or from an aperture of relatively large diameter, diffraction will result. It is obvious that if instead of plane waves passing through a hole a solid plate with parallel faces is set in vibratory motion in such a way that all points of its face are vibrating in the same phase, the same conditions will prevail. And if the plate has a diameter large in comparison with a wave-length and the plate is forced to vibrate in a fluid, a radiation will be produced and diffraction will take place forming the zones of maximum and minimum intensities. Such a vibrating disk may be formed of plates of any substance possessing the piezo-electric property.

One of the outstanding substances exhibiting the piezo-electric property is quartz cut from the crystal in a certain way. If a plate of it, appropriately cut, is sub-

jected to a mechanical pressure, positive and negative electricity will be liberated on opposite faces of the plate; and if there is a contractive force applied there will be liberated a positive charge on the previously negative face. Conversely, if a quantity of positive and negative electricity is applied to the two faces there will be a contraction or expansion depending on the direction of the applied electric field.

The above property is the principle of the kind of ultra-sonic transmitters used here. If an alternating electric field is applied to the opposite faces of the quartz, alternate expansion and contraction of the plate takes place and these vibrations are transmitted to the medium surrounding the transmitter.

The Ultra-Sonic Transmitter.

The quartz used for the ultra-sonic transmitters is cut from a quartz crystal in a definite manner. In a quartz crystal there are three electric axes. The piezo-electric phenomenon is exhibited both along, and in one direction at right angles to, these axes. In Figure 1, A_1, A_2, A_3 represent a hexagonal section perpendicular to the principal axis. A_1a_1, A_2a_2, and A_3a_3

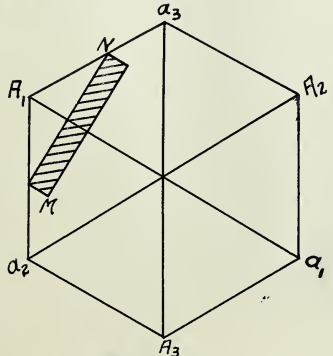

Fig. 1.

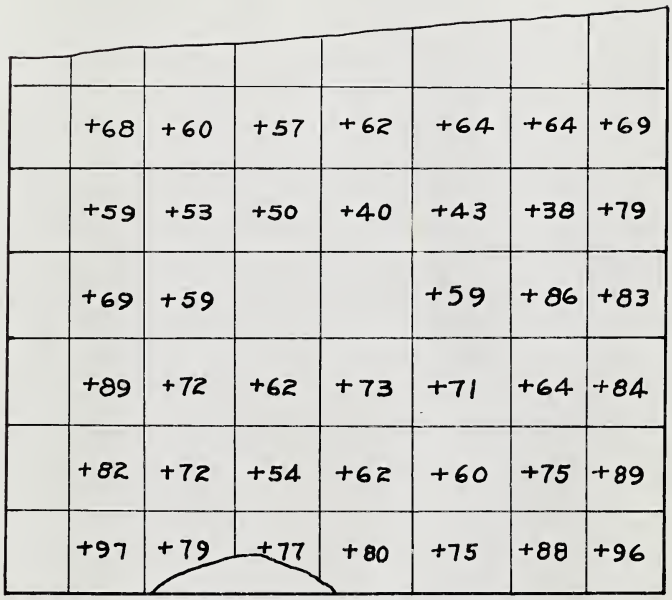
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FIG. 3.

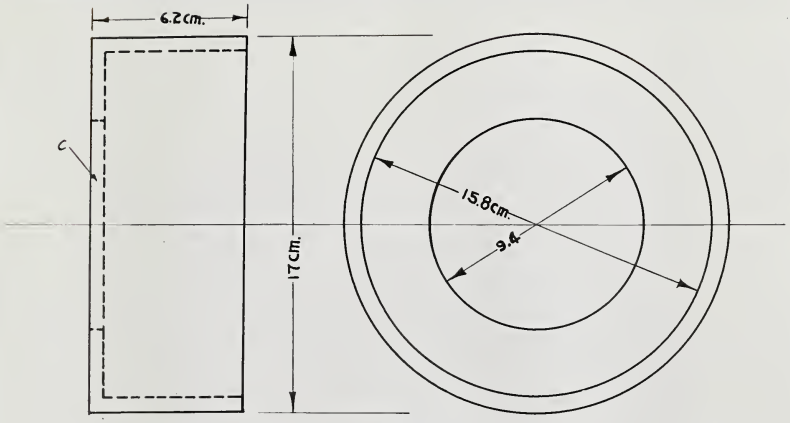


FIGURE 4

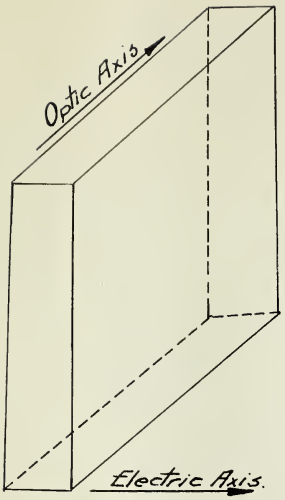


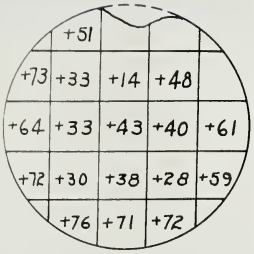
Fig. 2.

are the electric axes of the crystal and are at an angle of 60° to one another. If a rectangular prism MN be cut from the crystal with faces and sides perpendicular and parallel respectively to the electric axis A_1a_1, then this plate will exhibit the piezo-electric phenomenon. Figure 2 represents such a plate.

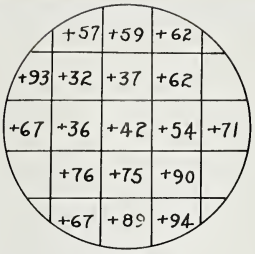
One type of transmitter used in this laboratory was built as follows: A solid rectangular piece of quartz, 1.508 cm. thick and surface dimensions 11.5 cm. x 10 cm. was cemented to a back steel plate 1.587 cm. thick and 9.4 cm. in diameter.

The plate was subjected to a test in order to find the electric properties of its faces. This test was carried out by an ordinary electrometer and a mechanical compression pressure applied to the faces. This pressure caused a release of electricity on the faces and the results for one face were as shown in Figure 3. The number in each section represents a comparative value of the electric condition of the quartz in that particular area.

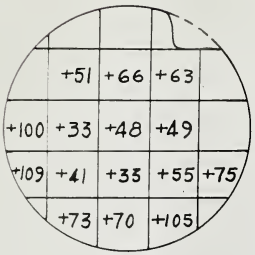
The quartz plate with a back plate of steel was placed in a metal casing, the shape and dimensions of which are shown in Figure 4, with the open face of the quartz covering the hole C.



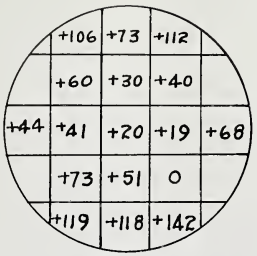
Disc 1.



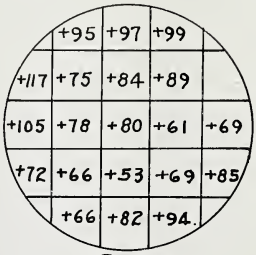
Disc 2.



Disc 3.



Disc 4.



Disc 5.

FIGURE 5

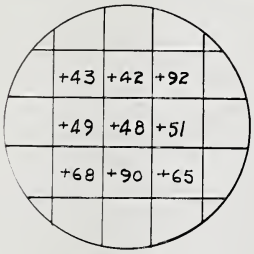


FIGURE 5A

Top of Pile Constructed.

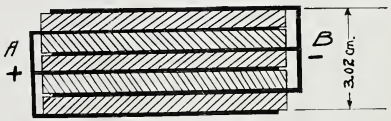


FIGURE 6

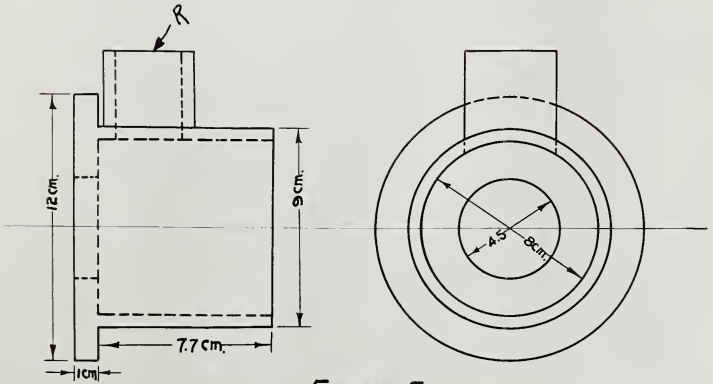


FIGURE 7

To the back plate was attached a well insulated cable. The casing was then filled with a hot mixture of paraffin wax and resin, in proportions of 1 to 3 by weight, and allowed to cool. If the instrument is placed in a medium like water, and alternating voltage applied to the two faces of the quartz, the quartz will be forced to contract and expand alternately and thus set up longitudinal oscillations in the water, transmitted in a direction normal to the face of the quartz.

The other type used was erected in the following manner: Five circular quartz discs shown in Figure 5 were placed as shown in Figure 6, positive faces together and negative faces together, with a 0.006 inch circular plate of zinc between each disc, the positive zinc strips connected together to form one terminal A and the negative zinc strips all joined to form the other terminal B. The quartz discs were subjected to a piezo-electric test as described above and the results are shown in Figure 5. When placing the discs in position a hot mixture of paraffin wax and resin was flowed between the discs and care taken to expel all air bubbles between the plates. The pile thus constructed was placed in a casing, the style and dimensions of which are shown in Figure 7. The discs rested against a thin brass plate, which in turn covered the 4.5 cm. opening. The negative terminal A was connected permanently to

Figure
ARRANGEMENT
of
GENERATING CIRCUIT

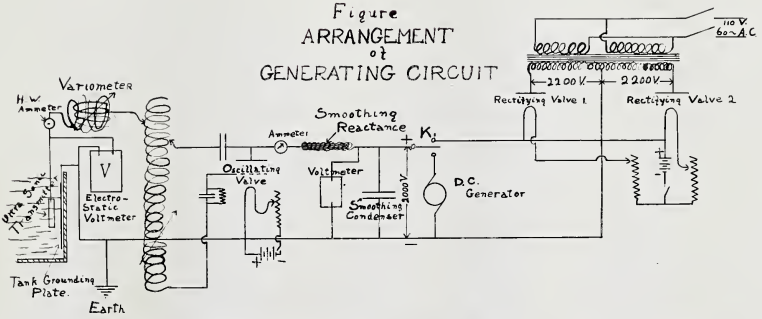


Figure
GENERATING CIRCUIT E.M.F.'S.

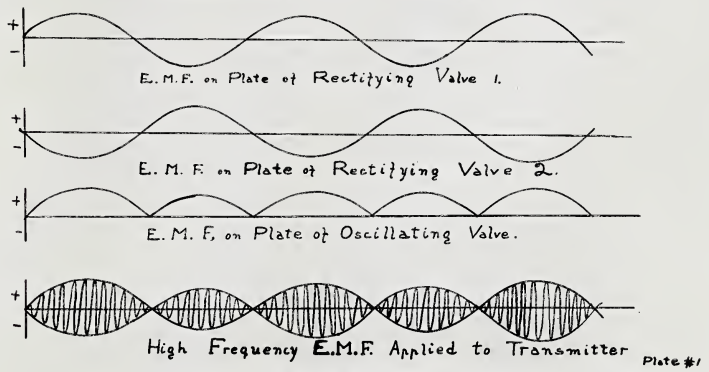

+
-
E. M. F. on Plate of Rectifying Valve 1.
E. M. F. on Plate of Rectifying Valve 2.
E. M. F. on Plate of Oscillating Valve.
High Frequency E.M.F. Applied to Transmitter

Plate #1

FIG. 8.

the casing and a well insulated cable was attached to the positive terminal B and brought out through the pipe opening R. When an alternating voltage was applied to the two terminals a small increase or decrease was produced in the thickness of the pile of quartz, depending on the direction of the electrostatic field.

Constructed as above these transmitters really form electric condensers with quartz as the dielectric. The transmitters were excited by means of high frequency oscillating voltages generated from electronic valves as in wireless, and electrical energy was thereby changed to ultra-sonic energy of the same frequency as the voltages applied to the plate. The generating system is shown in Figure 8. A 60-cycle, 110-volt A. C. supply was used as source of electrical power. This was stepped-up to 4400 volts and rectified by two 2-electrode valves connected in parallel as shown. The potential on the plate of the generating valve was, therefore, a series of uni-directional pulses giving a tonic train of 120 pulses per second. The oscillations thus given to the plate, and then to the water, were therefore not continuous, but were transmitted in a tonic train in which the group frequency was 120 per second. See Figure 8. The transmitter acted as a condenser of the oscillating circuit, one side of the plate being connected to the filament end of the oscillating inductance and thus behaving as one side of

a condenser, while the other side of the condenser was furnished by the insulated face of the plate. An electrostatic voltmeter measured the high frequency voltage across the transmitter.

A 2000-volt D. C. source was obtained by switching the key K. The D. C. source furnished power for the generation of continuous waves.

The arrangement and devices described above furnish a very convenient means of carrying out researches on wave and other problems which cannot be treated in any other way, or at least not with so great a facility and accuracy. For instance, the compressibility of liquids has been very little investiaged on account of experimental difficulties. A sufficient proof of the difficulty of obtaining accurate results is furnished in the comparison of the results of different observers. But with the use of ultra-sonic waves a new method is obtained, using apparatus of only ordinary and convenient dimensions, which affords the means of making accurate measurements. Similarly, viscosity and absorption effects may be more easily and accurately determined by this method. In fact the ease with which the ultra-sonic waves can be generated and controlled furnishes a powerful and reliable device to study the elastic properties of any medium in which such waves can be generated.

Audible Method of Detecting Ultra-Sonic Energy.

As mentioned above the ultra-sonic oscillations generated in the water, when excited by an alternating current source, were transmitted in a tonic train with a group frequency of 120 per second. On account of this condition it is possible to detect the ultra-sonic beam with a device such as a stethescope by listening for the 120 cycles per second frequency. The listening detector device used was made as follows: Two very thin mica discs, each 2.0 cm. in diameter, see Figure A, were supported by a brass ring R, 2 centimeters in diameter, 0.5 cm. thick with a 1.4 cm. concentric hole. A hollow brass tube B was soldered rigidly to the ring and a hole L connected the two spaces S and S_1. The tube B, about 30 cm. long, was connected at the other end to the ear tubes of the stethescope. When a sound wave impinges on the mica discs they are forced to vibrate, and the sound thus set up is carried to the ear by the tube B and the stethescope. This listening detector device if placed in the path of the ultra-sonic beam is subjected to the radiant pressure of the high frequency ultra-sonic waves. The beam is made up of a number of groups constituting the tonic train; and each group

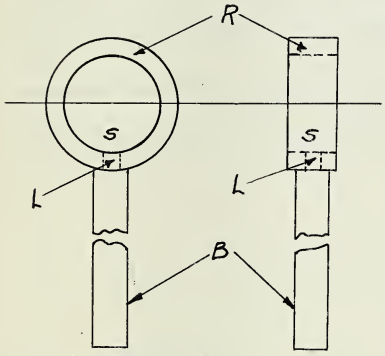


Fig. A.

is capable of exerting a pressure. This pressure is exerted on the discs in the following manner: each tonic train is made up of a number of oscillations and these oscillations are so rapid that the mica discs cannot respond, just as the diaphragm of a telephone receiver cannot respond to the rapid wireless oscillations of a reception circuit. The mica discs can respond for each group, since each group contains energy and thus exerts radiation pressure. The pressure causes a movement of the mica discs and a sound impulse is thus produced in the stethescope. The number of impulses per second so produced will be the same as the number of groups per second in the tonic train, which in the present case was 120. Consequently a note of 120 pitch is heard in the stethescope.

PART I (a)

SHAPE OF ULTRA-SONIC BEAM IN AIR.

Measurement of Energy Intensity of Radiation by Pendulum.

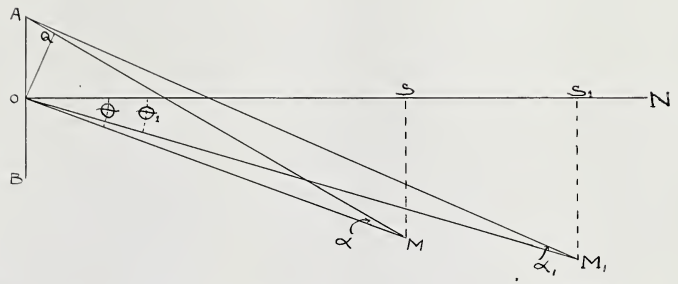
Lord Rayleigh' has shown that a plane sound wave impinging upon a perfectly reflecting surface produces a pressure of magnitude = 2 E, where E = energy density in the incident wave train; and W. Altherg" has used this radiant pressure to measure the absolute intensity of sound radiated by a Kundt's tube in air.

Ultra-sonic energy can be detected in various ways but in all quantitative methods the radiant pressure of the ultra-sonic waves is utilized. Perhaps the most successful quantitative method of detecting such energy is the torsion pendulum method.

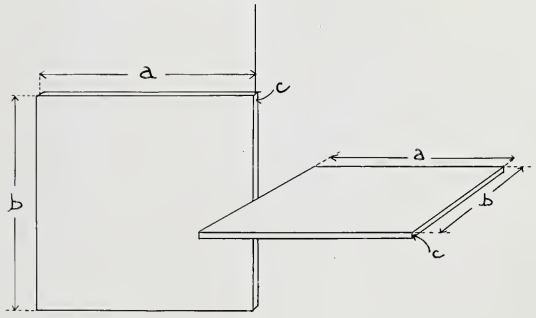
A torsion pendulum is made by soldering two thin, small, plates of brass, lead, or some other metal, square or circular in shape, at right angles to each other and suspending them by a phosphor bronze strip.''' Lead is more extensively

' Phil. Mag.(6) p. 338, 1902.
" Drude Annalen der Phys. 1903.
'''Lehmann: Ultra-Sonic Oscillations, Pt. I, Sec. 3(d).

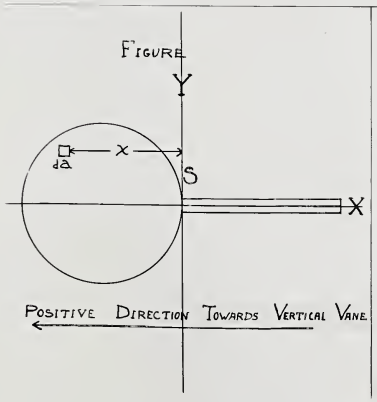
FIGURE



FIGURE



FIGURE



FIGURE

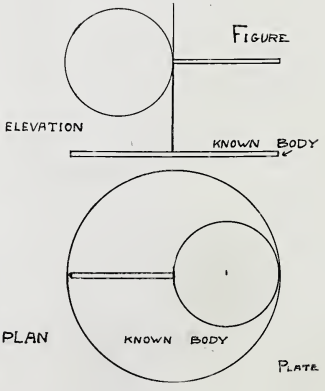


PLATE 25

FIG. 9.

used than other metals because of the low velocity of sound in this material; thus sheets of lead are suitable for the vanes. The pendulum is suspended in the ultra-sonic beam and the radiant energy of the beam striking the vertical vane deflects it. A counterpoise is furnished by the horizontal vane.

A plan and elevation view of the square and round pendulums used in this laboratory is shown in Figure 9. Various sizes have been used, but investigation has shown that the best conditions prevail when a circular vane pendulum is used, having a thickness of about 0.1 λ and a diameter of about 1.5 λ , where λ is the wave-length of the ultra-sonic energy in the material of the vane.

For purposes of investigating the properties of the beam the pendulum is passed through a vertical section and horizontal section of the beam. The pendulum is deflected by the radiation pressure of the beam and the deflection is reduced back to zero by twisting forward the suspension. The amount of torsion in the suspension is measured by noting the angle of twist turned through in order to bring the pendulum back to its zero position, that is, to the position where the vertical vane is at right angles to the direction of the energy beam. It can be shown mathematically that the amount of restoring torque produced by the torsion in the pendulum is

proportional to the energy density of the ultra-sonic beam.[1]

Ultra-Sonic Beam in Air.

For investigating the ultra-sonic beam in air, the pile of plates transmitter was used as a source of ultra-sonic energy. Attempts were made to obtain Kundt's figures in a tube containing lycopodium dust, by placing the tube along the axis of the beam. A reflector was placed approximately 60 cm. from the transmitter and at right angles to the direction of the beam. Different sized tubes were used but no figures were formed. The lycopodium dust was next allowed to fall through the beam and was caught on a glass plate placed at the bottom limit of the beam. It was thought that with proper adjustment of the reflector the advancing and reflected waves would meet to form standing waves and that the dust particles would be driven into the nodal lines out of the agitated regions and that the arrangement of the dust would be similar to the figures in a Kundt's tube. However, no results were obtained.

In the next attempt to study the beam a torsion pendulum was employed. A lead pendulum, 2 cm. in diameter by 0.25 cm. thick, was suspended by a phosphor bronze 0.0015 inch strip, 80 cm. long, and placed in a position to obtain a vertical and horizontal cross-section of the beam. On account of air currents and the very little damping of the pendulum it was

[1]Lehmann: Ultra-Sonic Oscillations, Pt. I, Sec. 3.

found necessary to shield the pendulum from air currents and to provide a damping arrangement. The latter was furnished by attaching to the lower end of the vanes by means of a small but rigid brass wire another brass wire at right angles to the supporting wire, 3.81 cm. long by .071 cm. in diameter, moving in a beaker of water. The transmitter was forced to oscillate at its approximate resonant frequency and the readings obtained are as in Table I.

TABLE I.

Temperature °C.	Frequency of Oscillations.	Voltage across Transmitter.	Position of Suspension.	Pendulum Deflection.
		Horizontal Section		
17.75	40,500	1500	4 Right	18
17.75	"	1525	2 "	24
18.3	"	1525	6 "	10
18.35	"	1550	8 "	3
18.4	"	1525	9 "	0
18.0	"	1500	0 "	31
18.0	"	1500	2 Left	23
18.0	"	1525	4 "	14
18.1	"	1510	6 "	12
18.2	"	1500	6 "	10
18.1	"	1525	8 "	5
18.2	"	1525	10 "	0

TABLE I (Continued)

Temperature °C.	Frequency of Oscillations.	Voltage across Transmitter.	Position of Suspension.	Pendulum Deflection.
		Vertical Section		
19.0	40,500	1530	1 Down	22
17.5	"	1525	1 "	27
18.95	"	1525	2 "	24
19.0	"	1530	3 "	21
17.5	"	1525	3 "	26
17.5	"	1525	6 "	9
16.4	"	1500	9 "	2
16.65	"	1540	10.5"	0
16.4	"	1500	12 "	2
16.65	"	1525	15 "	0
17.5	"	1525	2 Up	13.5
18.85	"	1525	4 "	8
18.75	"	1525	5 "	5
18.95	"	1525	6 "	4
18.75	"	1525	7 "	7
18.75	"	1525	9 "	3
18.75	"	1525	11 "	2
18.85	"	1525	13 "	0

Comparison with Verdet's Formula.

As mentioned above ultra-sonic oscillations generated by an oscillating disc are analagous, mathematically, to the optical phenomenon of a parallel beam of light passing through a small circular opening. E. Verdet' has mathematically treated the optical case.

'Lecons d'Optique Physique,
Tome 1, page 301.

Verdet made the assumption that parallel rays of light pass through a circular opening in a direction normal to the plane of the opening. Since the phenomenon is symmetrical, the effect will be the same in all planes which are normal to the opening and include a diameter of it. Verdet considered the effect in one plane. Let AOB, Figure B, be the opening and M the point at which the intensity of illumination is to be considered. Verdet has shown that

$$I^2 = \pi^2 R^4 \left[1 - \frac{m^2}{2} + \frac{m^4}{(2!)^2 3} - \frac{m^6}{(3!)^2 4} + \cdots \right]^2$$

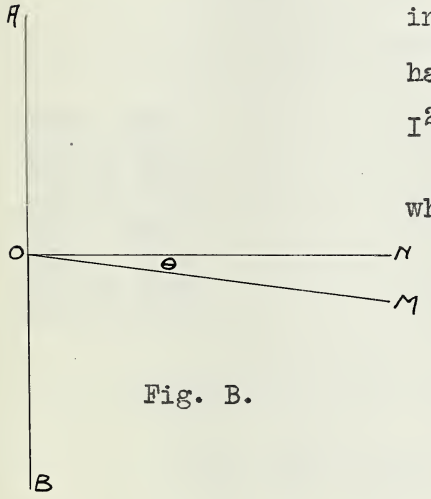


Fig. B.

where I^2 = intensity of illumination at M.

R = radius of opening.

$m = \frac{\pi R \sin \theta}{\lambda}$

θ = angle between the line joining M to the centre of the opening and the central axis ON.

λ = wave-length in the medium of the energy under consideration.

As θ increases, m increases, and the value of the bracketed expression becomes alternately positive and negative and therefore must pass through zero values. That is, as θ increases the intensity of illumination varies from a maximum when $\theta = 0$, down to zero intensity, then increases to a maximum and again falls to zero, and so on. Thus, the energy is dis-

tributed from the opening in zones of maximum intensity separated from one another by zones of zero intensity. The values which give maximum and minimum intensities have been calculated by Verdet and his results are found in Table II together with the relative values of various maximum intensities.

TABLE II.

	$\frac{m}{\pi}$	Intensity
First Max.	0	1.0
First Min.	0.610	0
Second Max.	0.819	0.0175
Second Min.	0.116	0
Third Max.	1.333	0.00415
Third Min.	1.619	0
Fourth Max.	1.847	0.00165
Fourth Min.	2.120	0

The above table shows that the second and subsequent zones have intensities which are negligible in comparison with the intensity of the first maximum. Thus, most of the energy is limited to the central zone, the angular width of which can be determined from the value m at the first minimum.

As found by Verdet the value of $\frac{m}{\pi}$ for the first minimum is 0.610. Since $m = \frac{\pi R \sin \theta}{\lambda}$ or $\sin \theta = \frac{m}{\pi} \frac{\lambda}{R}$, then $\sin \theta = .610 \frac{\lambda}{R}$ or $1.22 \frac{\lambda}{D}$ where D is the diameter of the

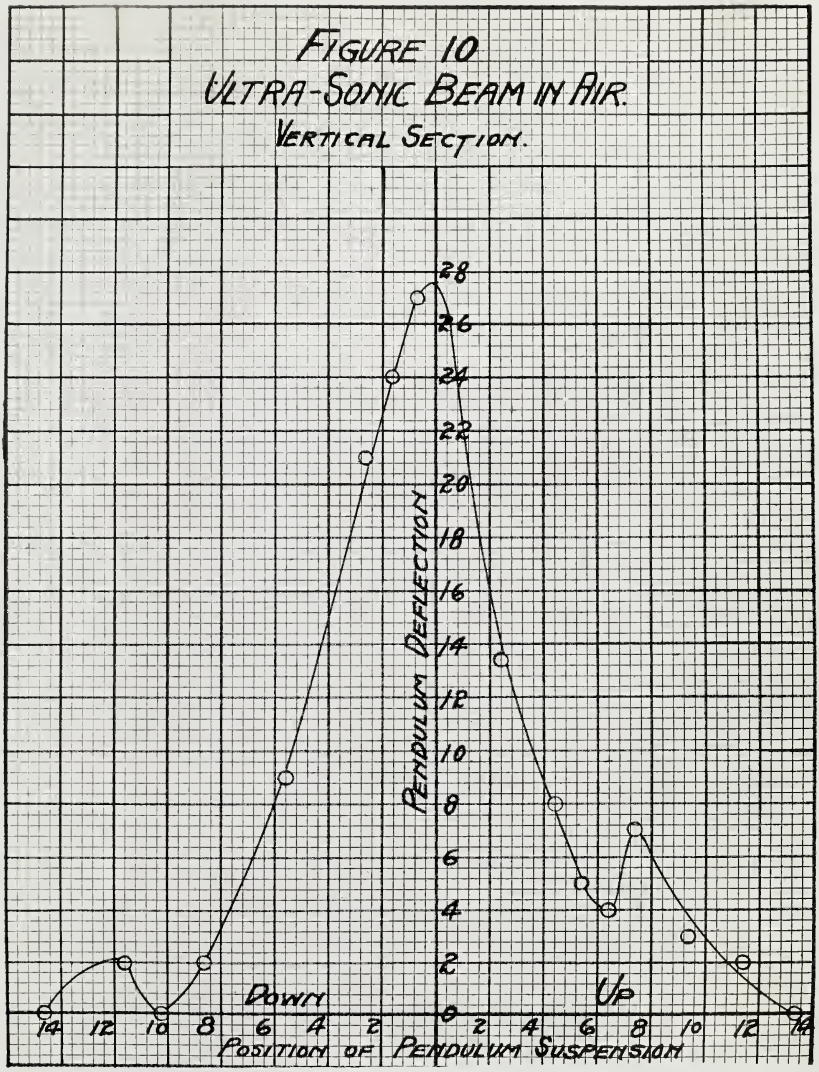


FIGURE 10
ULTRA-SONIC BEAM IN AIR.
VERTICAL SECTION.
PENDULUM DEFLECTION
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
DOWN
UP
14 12 10 8 6 4 2 0 2 4 6 8 10 12 14
POSITION OF PENDULUM SUSPENSION

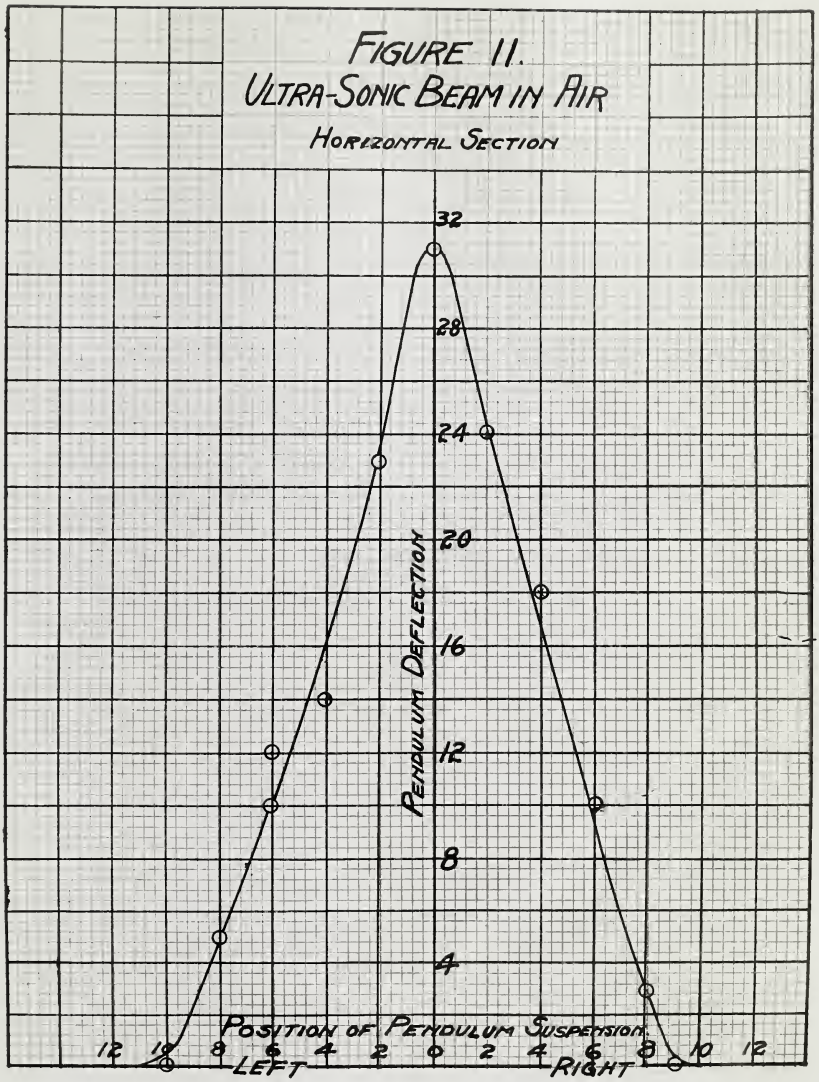


FIGURE 11.
ULTRA-SONIC BEAM IN AIR
HORIZONTAL SECTION
32
28
24
20
16
12
8
4
PENDULUM DEFLECTION
POSITION OF PENDULUM SUSPENSION.
12 10 8 6 4 2 0 2 4 6 8 10 12
LEFT
RIGHT

opening. Airy'has pointed out that in order to obtain the central zone, λ must be small in comparison with D.

The condition as investigated mathematically for optics by Verdet and Airy is obtainable in the case of ultra-sonic oscillations. When the transmitter used in the investigation of the ultra-sonic beam in air was placed 102.5 cm. from the pendulum, as described above, deflections were produced, the amount of the deflection being greatest when in the center of the central zone and decreased when moved away from that position. The energy density is proportional to the deflection. This transmitter had a diameter of face of 6", and oscillated at a frequency of about 40,900 cycles per second in air, at the temperature of 18.5° C. Taking the velocity of sound in air as 33,100 cm. per second at 0°C., then velocity at 18.5° C. is 34,210 cm. per second. The pendulum was moved through the central zone, vertically and horizontally, and the shape of the beam sections in each case is shown in Figures 10 and 11 respectively. Choosing the horizontal section, it is noted that the maximum energy density when $\theta = 0$ is represented by 31. Taking this as 1, then the relative value can be found of any point on the curve. For instance, to get the energy density represented by 0.9, take

'G. B. Airy: Mathematical Tracts,--Intensity of Light Passing Through a Hole.

0.9 of 31 and get 27.9. Looking for this value on the curve it is noticed that it has abscissa reading 1.2. Then the θ in radians will be 1.2/102.5 where 102.5 represents the distance from the transmitter. Different values of energy density are taken and the corresponding values of θ found. Figure 12 shows the energy density plotted as a function of θ in radians.

In Verdet's expression I^2 represents the intensity of illumination in the optical case and corresponds with the energy density of the ultra-sonic beam. As stated above the energy density is proportional to the pendulum reading; then to obtain a comparison between Verdet's optical results and those obtained experimentally for the ultra-sonic beam in air it is only necessary to compare the I^2 of Verdet's expression with the experimental ultra-sonic beam in air. The values of R and λ occurring in the ultra-sonic sections are substituted in Verdet's equation and the values of I^2 corresponding to different values of θ are calculated. The values of m which make $I^2 = 0$ are found by putting $\sin \theta = .61 \times \frac{\lambda}{R}$. In the calculation of the wave-length $\lambda = \frac{V}{n}$ is used where V = velocity of sound in air and n = frequency of oscillation.

In order to compare Verdet's case with the ultra-sonic case using the radius of the transmitter for the radius of the opening and the same frequency, the following calculation was made:

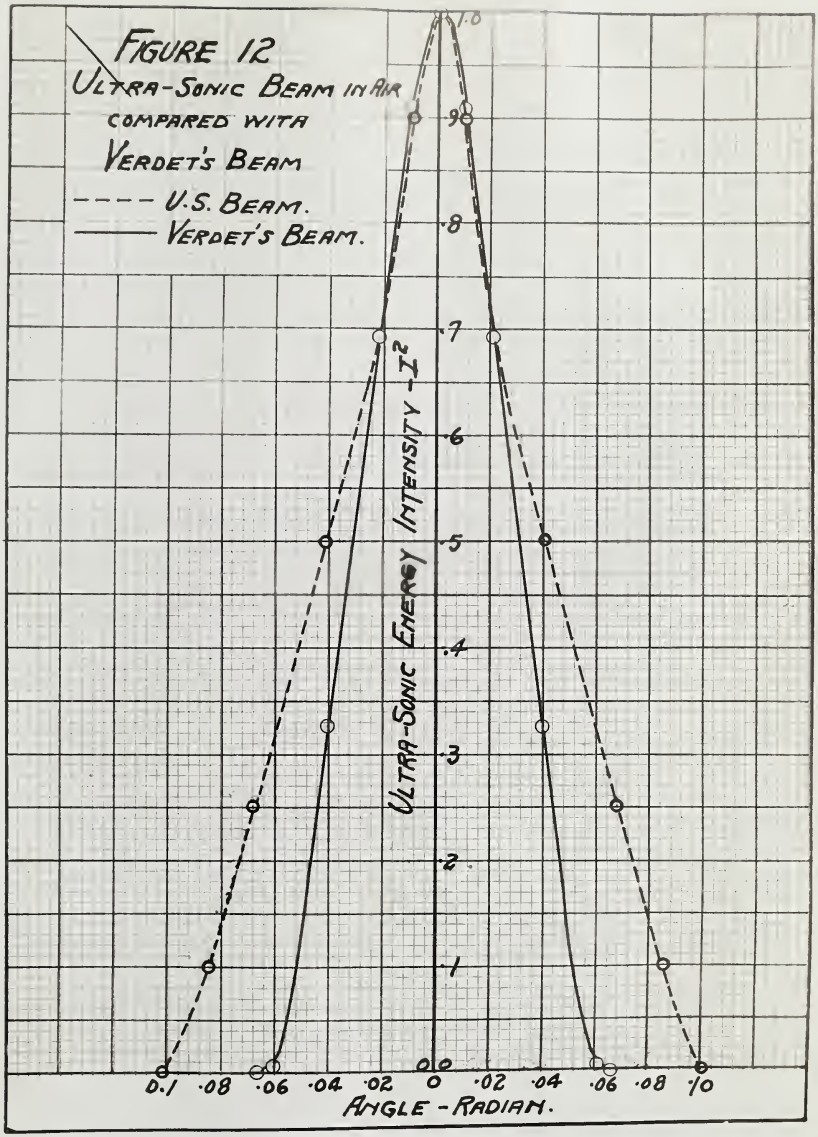


FIGURE 12
ULTRA-SONIC BEAM IN AIR
COMPARED WITH
VERDET'S BEAM
---- U.S. BEAM.
—— VERDET'S BEAM.
ULTRA-SONIC ENERGY INTENSITY - I²
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0.1 .08 .06 .04 .02 0 .02 .04 .06 .08 .10
ANGLE - RADIAN.

$$\lambda = \frac{34,210}{40,900} = .837 \text{ cm.}$$

$$R = 3.0 \text{ in.} = 7.62 \text{ cm.}$$

$$\text{Sin } \theta = .61 \times \frac{\lambda}{R} = .61 \times \frac{.837}{7.62} = .0670$$

$\therefore \theta = 3.8^{\circ}$ (approx.) which is the angle that will make the energy density $I^2 = 0$.

$$m = \frac{\pi R \sin \theta}{\lambda}$$

Different values of m are substituted in Verdet's formula

$$I^2 = K(1 - \frac{m^2}{2} + \frac{m^4}{(2!)^2 3} - \frac{m^6}{(3!)^2 4} + \ldots\ldots)^2$$

by choosing various values of θ from 0° up to 3.8°. When $K = 1$, the value of I^2 can be determined. The calculated values are found in Table III and Figure 12 shows the value of I^2 plotted as a function of the angle in radians.

TABLE III.

Distance from Transmitter = 102.5 cm.
Frequency of Oscillation = 40,900 cycles per sec.

Angle in Radians.	I^2
.0663	0.0
.0611	.0030
.0401	.3272
.0209	.6922
.0105	.9120
0.0	1.0

Comparison with Curve of C. E. Lane.[1]

In an investigation of "Minimum Sound Energy for Audition for Tone of High Frequency" Lane used the Hewlett Generator[2] as a source of high frequency sound energy. In his calculations Lane assumed the amplitude of the sound wave to be the same at the diaphragm surface as that of the diaphragm. The total amount of sound energy E given off per second by one side of the diaphragm is given by

$$E = \tfrac{1}{2} S \rho V A^2 w^2$$

where S = area of each side of the diaphragm

ρ = density of the air

V = velocity of sound in air

A = amplitude of vibration of the diaphragm

w = 2π times the frequency.

Quoting Lane: "Considering a simple spherical distribution, the energy density per sq. cm. at a distance D cm. from the source is $E/2\pi D^2$, which represents the average value for energy density on the surface of a hemisphere of radius D. If R is the ratio of the actual value, J, at a distance D from the source, to the average value, then,

$$J = ER/2\pi D^2$$

"Assuming the sound field to be symmetrical with respect to the axis of the diaphragm the relative distribution

[2] Phys. Review (2) XIX, p. 52, 1922.
[1] Phys. Review (2) XIX, p.492, 1922.

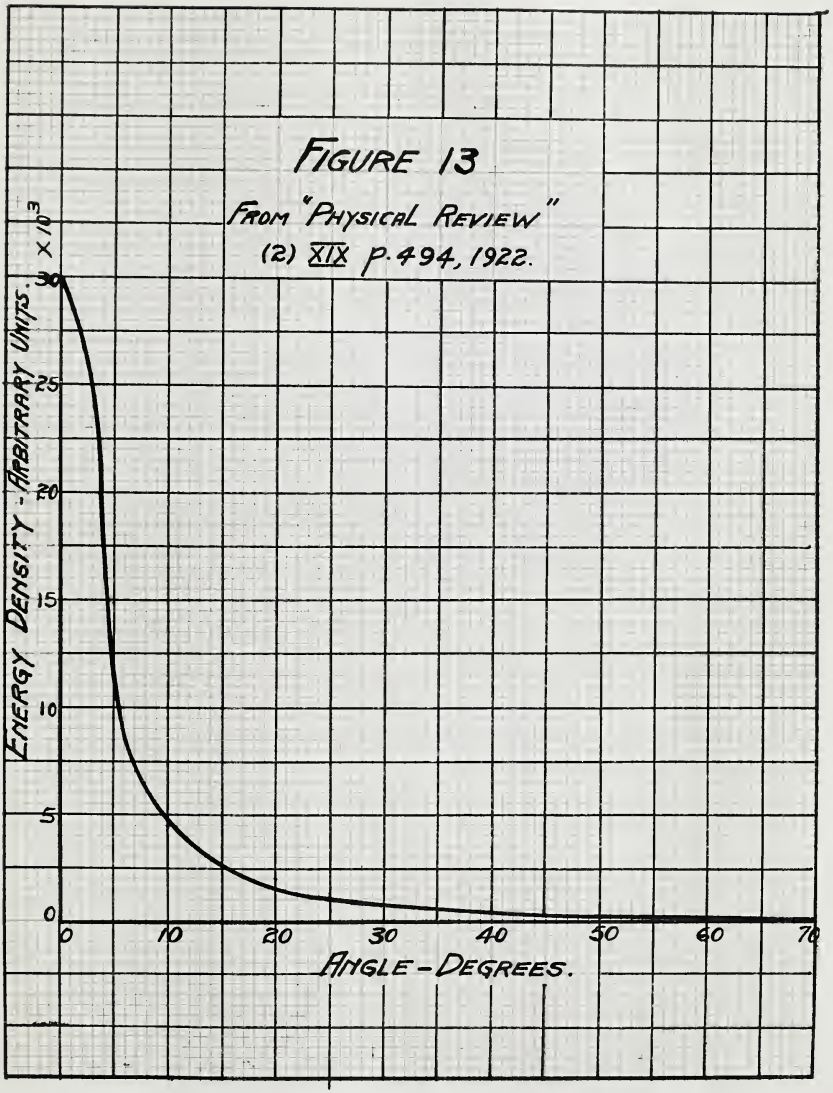


FIGURE 13
FROM "PHYSICAL REVIEW"
(2) XIX P. 494, 1922.
ENERGY DENSITY - ARBITRARY UNITS. × 10³
30
25
20
15
10
5
0
0
10
20
30
40
50
60
70
ANGLE - DEGREES.

of energy density on the surface of a sphere of radius D is determined by calculation from the measurement of a minimum audible amplitude of the diaphragm as it was turned about a vertical axis, through its center, through an angle of 180°." Figure 13 shows energy density determined at 10,000 cycles per second plotted in arbitrary units as a function of the angle.

In order to compare Verdet's optical beam with the beam formed by the Hewlett Generator in air the same method was followed as in the comparison of Verdet's optical beam and the ultra-sonic beam in air. The frequency for both = 10,000 cycles per sec. Experiments were carried out at night. Velocity of sound in air 33,900 cm./sec. $\lambda = \frac{V}{n} = \frac{33,900}{10,000} = 3.39$ cm. R = 5 cm. $\sin \theta = \frac{m}{\pi} \frac{\lambda}{R} = .610 \times \frac{3.39}{5.00}$ . .·. $\theta = 0.4259$ radian (approx.), the angle which gives zero energy density. Then substituting in Verdet's expression, the values of I^2 corresponding to chosen angles can be determined. The calculated results are shown in Table IV.

TABLE IV.

Frequency of Oscillation = 10,000 cycles per sec.
Distance from Transmitter= 150 cm.

Angle in Radians	I^2
.4259	0.0
.4014	.0001
.3002	.0958
.2007	.3950
.1012	.8590
0.0	1.0

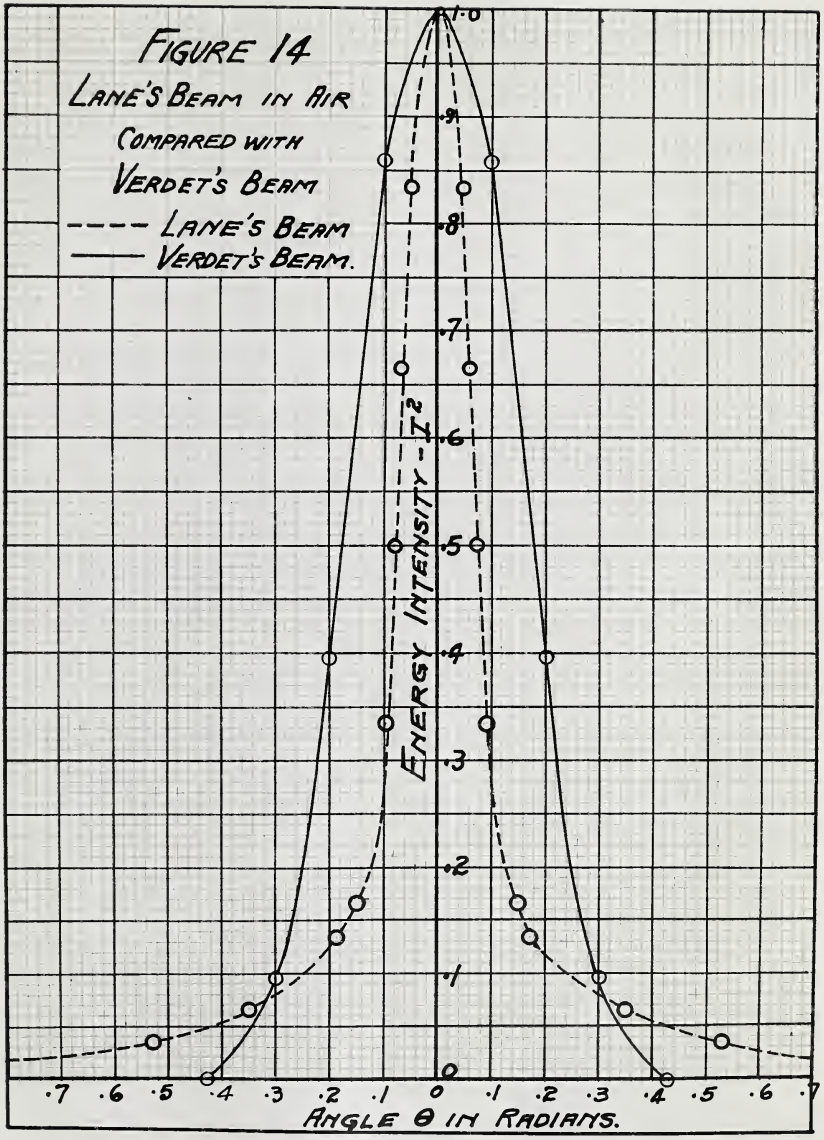


FIGURE 14
LANE'S BEAM IN AIR
COMPARED WITH
VERDET'S BEAM
---- LANE'S BEAM
—— VERDET'S BEAM.
ENERGY INTENSITY - I²
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
.7 .6 .5 .4 .3 .2 .1 0 .1 .2 .3 .4 .5 .6 .7
ANGLE θ IN RADIANS.

Lane did not make any comparison between his results and Verdet's expression, but using Lane's values and choosing 30,000 of his arbitrary units, the maximum energy density, as represented by 1, the relative values of energy density are known. Such values when plotted against the corresponding value of the angle θ in radians give a curve as shown in Figure 14. In the same figure is found Verdet's curve representative of optical values.

Thus it can be seen from figures 12 and 14 that both the energy beams obtained in air from a piezo-electric pile-of-plates generator and from a Hewlett generator do not conform to the mathematical relations deduced in Verdet's formula. But in Lehmann's work, already referred to, it was found that a piezo-electric oscillator, made in a certain way, when radiating into water generated an ultra-sonic beam in which the energy intensities agreed fairly well with the mathematical formula of Verdet. Possible explanations of this are the following:

First, when experimenting in air, the torsion pendulum cannot be absolutely free from air currents. The pendulum and transmitter were both shielded as completely as possible under the existing circumstances. However, air currents were still possible and on this account the shape of the beam may not be absolutely correct.

Again, the transmitter used for this experiment in air was made in such a way that the pile of quartz rested against a steel plate bearing on the face of the casing, a measured 6" diameter surface of the steel being exposed to the air medium. When the transmitter is generating ultra-sonic waves, the vibrating quartz and steel have perfect freedom only in an area in the middle of the opening in the face of the casing; the sides of the quartz very likely are impeded in their movement and the diameter that was chosen, namely, 6", is too large. This will introduce an error in the angle θ which in turn will change the values of all calculations depending on θ. Thus, the value of m is changed and consequently I^2 has different values from those shown and the shape of Verdet's beam will be wider. Also, it may be that the casing itself is forced into a vibratory motion by the oscillating quartz and the steel face plate, and this forced vibration interferes with the generation of the ultra-sonic sound waves and causes a change in the shape of the beam.

Verdet's beam does not show any similarity to Lane's beam in air from the Hewlett generator. A reason for this probably is that the confined edge of the vibrating diaphragm also restricts the motion of the diaphragm and the diameter used is, again, not the true diameter of the vibrating section. Also the resulting tone as given by the Hewlett generator is impure. The impurities consist of the octave and higher partials

of the fundamental. However, at high frequencies, 10,000and over, the impurities are inaudible unless the fundamental is very tense. At high frequencies the beam goes in a well defined beam, but as shown above it is not the beam given by Verdet's relation.

In the investigation carried out by Lehmann, it was found that there was a very good agreement in comparing Verdet's beam and the ultra-sonic beam in water.' The reason for good agreement probably is that the transmitter used by him was built in such a way that the hole of the casing exposed to the water was first covered with a thin sheet of mica, behind which was placed the quartz plate. The quartz plate was approximately the same size as the hole and thus there was no binding offered to the vibration of the quartz by the edges of the hole which supported the mica and quartz. Also the face of the casing was not set in motion by the vibrating quartz. These two effects were possibly present in the transmitters used in the experiment in air here described, and thus the disagreement with Verdet's relation.

'Lehmann: Ultra-Sonic Oscillations, Pt. I, Figure 29.

PART I (b).

AUDIBLE BEATS FROM INAUDIBLE SOURCES.

When two sounding bodies placed near one another have not exactly the same frequency, there will be interference between the waves emitted by them and the disturbance at any chosen point is continually changing. When compressions from both sources arrive simultaneously, or rarefactions arrive simultaneously, a maximum of sound is produced. In a short time condensations from one body arrive at the same time as the rarefaction of the other, and a minimum of sound results. Thus, there is set up a pulsating effect known as beats.

Suppose that two transmitters a short distance apart and facing each other are caused to oscillate with the same frequency; then two beams of ultra-sonic waves of the same frequency will be transmitted to the medium, and although the vibrations of the two transmitters may not be in step and there is interference between the waves, still at any given place the nature of the disturbance remains constant. But, if the two superimposed waves have different frequencies then the phase relation between the two will be constantly varying. The wave with the lower frequency will lag more and more behind the one

with the higher frequency until the phase difference between the two reaches 180°. After this the phase difference will get less and less until they are once more in phase. Thus the amplitude will be constantly varying from a maximum to a minimum, causing "beats", and the variation in the amplitude of the resultant motion will have a definite "beat frequency" equal to the difference between the two primary frequencies. For example, if the frequencies of the two superimposed ultra-sonic waves are 100,000 and 101,000 per second, respectively, the pitch of the resultant beat will be 1,000 cycles per second. With the separate frequencies the waves on striking the ear would produce no sound, since the adult ear cannot appreciate a sound with a frequency of over 20,000 per second (about); but with the superimposing of the waves a beat note with a frequency of 1,000 should be heard. Thus, we should expect to be able to produce beats from inaudible ultra-sonic sources.

In this part of the experimental work two transmitters excited from two different sources were placed 60 cm. apart in water and facing each other. One transmitter was forced to oscillate at approximately 40,000 cycles per second and the other at 40,000 ± 0 to 10,000. To investigate the nature of the effect between the two transmitters a listening device was made consisting of a small, thin, rubber nipple, attached to one end of a hollow brass tube the other end of which was attached to a stethescope. As stated above when the source of energy exciting

the generating valves is rectified alternating E. M. F., the ultra-sonic oscillations are transmitted in a tonic train, the group frequency being twice the number of cycles per second. See Figure 8. When the nipple is moved through the energy beam, the radiant pressure of each group of ultra-sonic oscillations impinges on the nipple causing a slight distortion, and a sound impulse is produced in the tube attached to the nipple.

In the present instance when the electrical exciting source was alternating the tonic train produced an audible note of 120 cycles pitch, which note was heard in the stethescope when any one of the two transmitters was set oscillating. When both transmitters were set oscillating, being 60 cm. apart and facing each other, and when the nipple was placed in the path of both beams, immediately musical beats were detected, superimposed on the note of the tonic train. The pitch of the beat note could be made to vary throughout the whole musical scale by varying gradually the frequency of oscillation of one or other of the transmitters, exactly the same as is done in the case of "heterodyne beat note" in wireless.

When the electrical exciting source was a continuous high potential, there was of course no tonic train and nothing could be heard in the water when one transmitter only was acting. But with the two transmitters acting, the beat note broke out and was easily heard as a pure musical tone. The above experiment was repeated at different times with both A. C. and D. C. electrical exciting sources, at primary frequencies from 40,000 to 150,000.

In these experiments indications of the formation of standing waves could also be noted. As the listening device was moved along the interfering beams there were noted maximum and minimum points of beat sound intensity. These maximum and minimum points were formed as is usual in sound; there was interference between the two sets of oppositely advancing waves. The waves could be continuous as in the case of D. C. excitation, or in a tonic train as in the case of A. C. excitation. Since successive maximum points or successive nodal points are one half wave length apart, the length of the wave is known. The frequency was known from the Hertzian wave meter, therefore the velocity of the ultra-sonic waves can be determined from the relation $V = n\lambda$. No readings of half wave lengths were taken in these experiments, but the method for determining the velocity of ultra-sonic waves in a medium was disclosed and the work in the velocity determinations following was a direct result of this experiment.

Attempts were made to detect the formation of beats in air but with no success. The failure was due no doubt to the insufficient radiant energy to impress the rubber nipple. The energy of the wave in water or a similar medium was great enough to distort the nipple and produce the necessary beat sound.

In the above if we consider two sets of plane waves advancing through a medium with frequencies of n_1 and n_2, then, expressing relations in two dimensions only, they may be represented thus: $y = a_1 \cos 2\pi(n_1 t - \frac{x}{\lambda_1})$ and $y = a_2 \cos 2\pi(n_2 t + \frac{x}{\lambda_2})$

respectively, where t = time, λ = wave-length, and x = distance measured along an axis parallel to the directions of the propagation of the waves.

When $n_1 = n_2 = n$, and a_1 and a_2 are about the same and equal to a, standing waves are formed by the superposition of the wave trains and such waves are represented by the expression:

$$y = a\left[\cos 2\pi\left(nt - \frac{x}{\lambda}\right) + \cos 2\pi\left(nt + \frac{x}{\lambda}\right)\right]$$
$$= 2a \cos\left(2\pi \frac{x}{\lambda}\right) \times \cos (2\pi nt).$$

When x = 0 or is an integral number of half wave-lengths, the amplitude of vibration is maximum and consequently an anti-node occurs. And when x is equal to an odd number of quarter wave-lengths, the amplitude of vibration is zero and a node results.

When n_1 is not equal to n_2 beats are formed and an interference pattern as well is formed which has a mathematical expression much more complicated than the formula for standing waves. But to the listening detector device there appears the usual form of standing waves. This result is due no doubt to the device having a width which will not allow it to separate the contiguous parts of the interference pattern because of their proximity.

We have an example of the corresponding case of standing waves in optics in the system of color photography devised by Lippmann.' The film of a photographic plate is placed in contact with clean mercury, and the plate is exposed to light on the glass side. The incident and reflected waves in the gelatin

'Edser: Light for Students, p. 423.

combined to form stationary waves, and if π represents the phase change on reflection, nodes will be formed at the reflecting surface and at distances $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$ .... from the surface. Then on account of the light producing a chemical change only at the antinodes, equidistant planes in which chemical change is produced were separated by spaces in which no chemical action took place, that is, at the nodes. When the film was developed and a cross section of it examined under the microscope, there were found a number of transparent layers of gelatin separated by thin layers of silver. The thickness of each transparent layer was equal to half the wave-length of the light used in the experiment.

Such experiments are extremely difficult to perform on account of the smallness of a wave of light. But in the case of ultra-sonics there is great ease in demonstrating all the properties of stationary waves on account of the very convenient size of wave, and the possibility of using the most ordinary materials.

PART I (c).

METHODS TO DETERMINE THE VELOCITY OF ULTRA-SONIC WAVES IN FLUIDS.

In the endeavor to determine the velocity of ultra-sonic waves in water, various kinds of experiments were performed. The results of some of the preliminary ones though inconclusive at present are of use in that they open up certain problems for further investigation.

The electrical device for generating the alternating voltage which is to be applied to the transmitter described above, emits electromagnetic waves which can be detected by a Hertzian wave meter calibrated to give readings of impressed frequencies to within 1.5 per cent. The electromagnetic waves thus given out are identical in frequency with the mechanical waves sent out by the transmitter; therefore, if the frequency of the electrical wave is known, the frequency of the mechanical wave is also known. If any method furnishes a means of determining the wave length of the mechanical waves, the velocity is known from the relation: velocity of a wave = wave-length x frequency of wave. Therefore efforts were made to devise

methods whereby such wave-lengths could be easily and accurately determined.

In the experiment on beats, already mentioned, in which standing waves were first noticed, a method for determining the velocity of ultra-sonic waves became apparent. It was noticed that as the nipple of the listening device was moved through the beam in a direction perpendicular to the transmitters that successive points of maximum and minimum intensities were heard, giving evidence of the formation of standing waves. Since the distance between two successive maximum planes or between two minimum planes is one half wave-length the wave-length could be determined. Thus the method for determining the velocity is obvious.

In the next experiment one transmitter only was used for the source of ultra-sonic waves, the other transmitter being inactive and used only as reflector for the ultra-sonic waves from the first. Then finally it was replaced by a lead plate reflector. In all cases standing waves were easily detected.

Application of Kundt's Tube Method and Results.

The first attempt to take actual readings for the determination of the velocity was made with a Kundt's tube, 61 cm. long and 3.4 cm. in diameter provided with an iron plunger 3 cm. thick, placed in the centre of the ultra-sonic beam. The position of the plunger could be adjusted in the

same way as the reflecting disc of the ordinary Kundt's tube. On account of coal dust being used in previous experiments in this laboratory' it was deemed advisable to proceed at first with this material as indicator.

The tube was placed in the beam of energy coming from a transmitter, all in water contained in a large tank. It was expected that if standing waves were formed the dust would settle at the nodal points in a fashion similar to that in which the lycopodium dust settles in a Kundt's tube. However, no figures were formed.

The plunger was removed and a reflector placed about 30 cm. from the transmitter and normal to the path of the beam. But again the experiment was unsuccessful, the dust figures were not formed. This tube was dispensed with and another tube 14.9 cm. long and 0.85 cm. in diameter was used. Dust figures were formed as soon as the tube was placed in the energy path. Tubes of various lengths and diameters were experimented with, but though a large number of readings were taken, no satisfactory results were procured.

It wasnoticed that the dimensions of the tube had a marked effect on the dust figures. In the process of using tubes of varying lengths and diameters it appeared that standing waves of much greater wave-length than those required for ultrasonic frequencies were superimposed on the existing ultra-sonic

'R. J. Lang: High Frequency Vibrations and Elastic Modulus of Metal Bars.

waves, and the position and size of the superimposed waves seemed to depend on the dimensions of the tubes used. And the figures were formed even when the tube was held at right angles to the direction of the beam. These results have been reserved for future investigations, but they are not published here.

Observations of Other Investigators Using Kundt's Method.

A publication by Karl Dorsing' refers to the determination of the velocity of sound in water when using a modification of Kundt's tube. In his first attempts he met with difficulties in determining the velocities, which difficulties were subsequently removed when the resonant vibrations of the walls of the tube were considered. He found that in order to obtain Kundt's figures it was necessary to excite the source of sound in exact resonance with the tube used, for if the period of oscillations of the glass and of the water at the point of contact were different the water oscillations could not produce the dust figures.

Dorsing' gives a formula for determining the velocity of sound in a limited mass of liquid which applies in particular cases.

$$u = \frac{v}{\sqrt{\left(1 - \frac{2\rho r v^2}{dE\left(1 - \frac{5\rho}{6r}\right)}\right)}}$$

where u = velocity of sound in the limited mass of liquid,
v = velocity in the tube,
r = radius of tube,
d = thickness of wall of tube,
E = elastic coefficient of the tube,
ρ = density of the liquid.

'Annalen der Physik IV, 25, p. 227.

The wave-lengths of the superimposed figures in the present investigation seem to indicate the presence of resonant vibrations with frequencies much lower than the ultra-sonic frequencies. This effect may be produced by resonant vibrations occurring in the tube walls as suggested by Dorsing's work or by low frequency, resonant, oscillations in the water column.

Lamb' has shown that the yielding of the walls produces an appreciable effect; the wave-length is lowered. If C_o be the theoretical velocity of sound in a liquid, and C the actual velocity, it is found that in the case of tubes with small wall thickness h that

$$C^2 = \frac{C_o^2}{1+2\,Ka/hE}$$

where a = internal radius

K = volume elasticity of the liquid

E = value of Young's Modulus for the material of the tube.

In the case of water where $K = 2.22 \times 10^{10}$, contained in a glass tube for which $E = 6.03 \times 10^{11}$ and whose thickness is 0.1 of the radius, $C = 0.759\ C_o$. When the walls are very thick $\frac{C^2}{C_o^2} = \frac{\mu}{K+\mu}$, where μ = rigidity. Since μ for glass is approximately 10 times K for water a diminution of about 5% in the wave-length would result.

No further attempts were made to determine the ultra-sonic velocity by the above method.

'Lamb: Dynamical Theory of Sound, p. 173.

Velocity Determination by Listening Method.

The transmitter was now placed 60 cm. from a reflecting wall. The listening detector device, consisting of the mica-walled capsule attached to a stethescope as described before, was fastened to a wooden frame and could be moved horizontally between the transmitter and reflector. Adjustments of the position of the detector were made until distinct maximum and minimum points were noted by the maximum and minimum sounds in the stethescope. As already described these maximum and minimum points forming the standing waves were produced by the meeting of the advancing and reflected tonic trains of ultra-sonic waves. The listening device was moved through a known distance and the number of maximum points noted. From this the number of half wave-lengths were obtained and the wave-length determined. The electrical frequency was read from the Hertzian wave meter and attached curve. This was also the frequency of the ultra-sonic wave. The velocity of the ultra-sonic wave in water was then calculated from the relation $V = n\lambda$. The readings with the calculated results are given in Table V. The results were satisfactory for they gave an idea of the approximate value of the velocity required.

TABLE V.

Temperature °C.	Wave-Length cm.	Frequency of Oscillation ~/sec.	Velocity cm./sec. x 10^5
15.35	3.70	40,700	1.50
15.35	3.446	43,700	1.50
14.85	3.104	48,000	1.49

N. B. The values of the wave-length given in Tables V, VII, VIII, IX, and X, are the average values of a number of readings.

FIG. 15.

The "Dust Figure" Method.

Up to this point a large body of water constituted the medium, but thereafter a much smaller body was used. A small wooden tank, 4' x 10" x 12" deep, was used with a glass window running the length of one side of the tank. A lead plate 0.25 cm. thick and of cross-sectional dimensions slightly less than those of the tank was used as a reflector. The mica disc listening detector device already described served to investigate the effects in the ultra-sonic beam, together with another method, which hereafter will be called the Method of the "Dust Figure."

If a fine dust be sprinkled on the surface of the water in which standing waves are formed, the particles as they pass through the energy field will be driven into nodal regions and swept out of those more agitated. Placing horizontally a flat rectangular piece of glass, or tray, at the bottom of the energy beam, these dust particles fall in curtains in the regions of minimum disturbance, and during the action the falling particles form parallel curtains plainly visible through the glass side of the tank. They settle in lines on the tray, which lines are the trace of the nodal planes. Figure 15 shows such an arrangement. Since the nodal planes are one half wave-length apart, if the frequency is known the velocity can be determined.

Different dust materials were used for this experiment, including coal cinders, coal dust, silt, cement and powdered

sodium carbonate. Coal cinders sifted through a 40-mesh sieve and caught on a 60-mesh sieve were found to give the most satisfactory results when the liquid used was water.

Velocity Determination Using a Smaller Quantity of Liquid.

In the experiments to date these two methods have been employed to determine the velocity of ultra-sonic waves in liquids, namely, (1) the listening method, and (2) the dust figure method. In the experiments when water was used, or a solution with water as a solvent, the quantity of liquid did not have to be considered. However, if different liquids are to be used, a method is necessary in which only a small amount of the liquid is required, for the sake of economy. And further, the dust in the dust method tends to make a liquid murky after three or four trials, so that the first attempts were made only with the listening device.

In the experiments where only a small quantity of liquid was available, a small metal tank of the following dimensions was built: 76.5 cm. long x 6 cm. wide x 8 cm. deep with a bay at one end, 14 cm. square x 12 cm. deep. A pile-of-plates quartz transmitter was placed in the bay and a plate of steel, 1 cm. thick, acted as a reflector of the waves.

Experiments were performed to check up the difference, if any, between the velocities as determined using a larger body of the liquid and those that were determined using the smaller tank. The transmitter oscillated at approximately

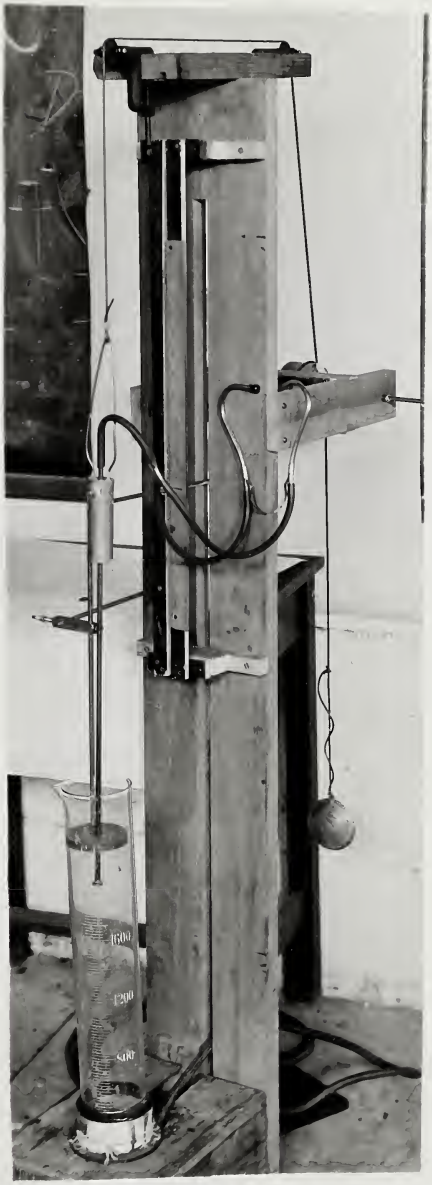


FIG. 16.

80,000 cycles per second and dust was allowed to fall through the energy beam. Velocities thus obtained for water varied from 1.5×10^5 cm. per sec. to 1.80×10^5 cm. per sec. But in the majority of cases the velocity was in the neighborhood of 1.5×10^5. In one case distilled water was used and the average velocity was 1.50×10^5. The results indicate that if the width of the beam is small the results are approximately the same as those obtained using the larger body of water. But it must be noticed that if the beam is wide enough to strike the tank sides or if the side beams from the transmitter strike the tank sides, reflections into the main beam cause a complication and distortion. The arrangement of the dust figures will not then be regular, and thus an error is possible in determining the distance between nodes. However, it was easily seen in the experiments when conditions were unsuitable for reliability.

Method Using a Vertical Column with Listening Detector Device.

An attempt was made to design an apparatus to meet the requirements stated above, namely, a small body of liquid, and at the same time avoid the use of dust particles. A cylindrical glass jar, 40 cm. long and volume of 2,000 c. cm. had rigidly attached to one end the small transmitter, and the whole was placed in a vertical position. See Figure 16. The hollow brass tube AB, with a mica diaphragm fixed at one end free to vibrate and the other end connected with the stethescope, could be lowered and raised gradually through the vertical column of

liquid. A pointer attached to the holder moved over a fixed scale. The liquid surface was the reflector. Readings were obtained using tap water. It was observed that the surface of the liquid changed slightly in position; for as the listening tube was pushed further into the liquid, more liquid was displaced and the surface was raised thus changing the position of the reflector and the position of nodes and loops. Also the motion of the surface disturbed the regularity and symmetry of the standing waves. Therefore, it was advisable to discard the surface of the liquid as a reflector and instead use a solid metal reflector so placed that it would always be completely immersed. Different lengths of columns were chosen by varying the position of the reflector.

In one set of experiments a saline solution of 1.025 density was used as liquid and velocity obtained using various length of columns. The maximum and minimum points were clear and distinct and the results obtained very satisfactory. From 1200 to 1400 c.c. of liquid were found to give the best results. See Table VI.

TABLE VI.

Temperature °C.	Frequency of Oscillations.	Voltage across Transmitter.	Wave-Length.	Velocity x 10^5
14.3	43,400	4,200	3.66	1.58
13.4	45,000.	3,450	3.53	1.59
13.7	44,800	4,200	3.60	1.61
15.4	45,700	4,700	3.42	1.56
15.6	45,300	4,500	3.45	1.56

When a 95% solution of ethyl alcohol was the liquid under consideration it was found that the regularity of the loops and nodes had disappeared. In the liquid column a very few, two or three, loops could be detected in order, then would follow a group similarly placed, but furnishing an arrangement different from the first group. Then would come another group similar to the first, then a fourth similar to the second. In certain arrangements an audible whistle note was easily heard.

There are two possible, obvious, causes for the above discrepancies. First, electrical harmonics in the oscillating circuit, and second, mechanical harmonics of the transmitter. Further readings were procured and attempts made to locate the electrical harmonics. The readings were similar to the above-mentioned set, and the work brought the undoubted conclusion that harmonics interfered with the regular arrangement of standing waves. Electrical harmonics, when they were present, could be detected by the wavemeter. If there are harmonics, either electrical or mechanical, then there will be in the column complicated interference of sets of advancing and reflecting waves; that is there will be a superposition of sets of waves giving an arrangement as noted. However, if these harmonics are eliminated there will be the regular interference of advancing and reflected waves, giving the regular form of standing waves. The conclusion is that if the electrical generating device is adjusted, or an electrical filter used, so as to give a pure note,

and if mechanical harmonics are also eliminated, then this method of a short vertical column of liquid with a listening tube is quite suitable to determine the velocity of the waves. The readings obtained with the saline solution were satisfactory because no harmonics were present and a pure note prevailed. The transmitter used had a resonant frequency of approximately 90,000; using a frequency lower than 70,000 it was noted that the effect of harmonics became apparent.

The method is not so reliable as the dust figure method, for, on account of having to locate the nodes and loops by a listening device, the holder of which occupies a space of over 1 cm., it is obvious that the frequency must be relatively low in order to obtain a half wave-length that will be large enough to make the error in measurement small.

But even in the dust figure method, if the beam is wide enough for intense portions of it to strike the sides of the tank, or if the side beams strike the sides there are reflections into the main beam from the sides of the tank, which produce an irregular and distorted arrangement of the maximum and minimum regions of sound intensity, and lead to irregular measurements of the distance between successive nodes or loops. For satisfactory results when using this device, the following conditions must prevail: the frequency of the wave must be low; there must be no harmonics either electrical or mechanical; and the beam must be narrow.

FIG. 17.

FIG. 18.

Figure 17 gives an idea of the arrangement of the dust particles when measurements are not taken, conditions for obtaining stationary waves being poor. Figure 18 shows the arrangement when measurements are taken. Generally the arrangement was much better than the figure indicates.

PART II.

MEASUREMENT OF VELOCITY

AND

ADIABATIC COMPRESSIBILITY OF FLUIDS.

The ease and accuracy with which ultra-sonic waves are generated and handled enables the methods described in Part I (c) to be used effectively, first, to determine the velocity of ultra-sonic waves in fluids, and second, to determine the adiabatic compressibility of fluids.

Measurement of Ultra-Sonic Waves in City Tap Water.

This measurement was carried out in a large tank of water. Table V, page 37, shows the readings and calculated results.

Change of Velocity with Change of Temperature.

Experiments were carried out to show the change in velocity of ultra-sonic waves with change of temperature of the medium. Both the listening and dust figure method were employed. City tap water was used in a tank 4' x 10" x 12" deep. The readings and results of this experiment are found in Table VII. Figure 19 represents the velocity plotted as a function of the temperature.

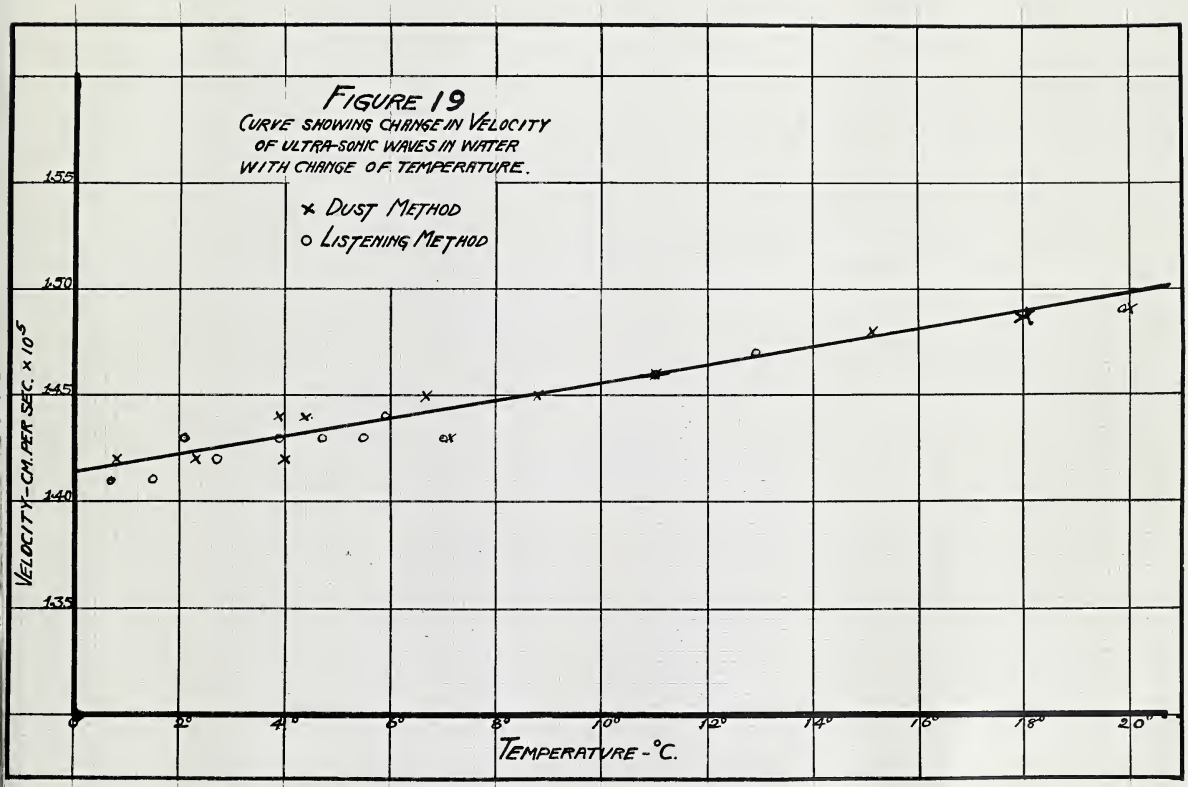


FIGURE 19
CURVE SHOWING CHANGE IN VELOCITY
OF ULTRA-SONIC WAVES IN WATER
WITH CHANGE OF TEMPERATURE.
× DUST METHOD
○ LISTENING METHOD
1.55
1.50
1.45
1.40
1.35
VELOCITY - CM. PER SEC. × 10⁵
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
20°
TEMPERATURE - °C.

TABLE VII.

Listening Tube Method.

n = frequency of oscillation.
V = velocity of ultra-sonic energy.
λ = ultra-sonic wave length.
N = maximum number of successive nodes measured.
L = distance occupied by N half wave lengths.

Temperature °C.	n cycles/sec.	N	L cms.	Average λ cms.	V cms./sec.
0.7	80,000	35	30.6	1.75	141,000
1.5	79,500	36	32.0	1.77	141,000
2.1	79,500	30	26.8	1.79	143,000
2.7	79,500	27	24.1	1.79	142,000
3.4	79,500	20	18.0	1.80	143,000
3.9	79,500	30	27.9	1.80	143,000
4.65	79,200	33	28.8	1.80	143,000
4.85	79,700	20	18.0	1.81	144,000
5.45	79,200	41	37.0	1.80	143,000
5.9	79,700	36	32.4	1.80	144,000
7.0	79,200	40	35.9	1.81	143,000
11.0	79,200	40	36.6	1.83	146,000
14.9	79,200	16	14.8	1.86	147,000
19.9	79,700	40	37.5	1.87	149,000
		Dust Figure Method			
0.8	79,500	41	36.9	1.78	142,000
1.75	80,200	20	17.5	1.75	140,000
2.3	79,500	30	27.1	1.80	143,000
3.9	79,500	27	34.4	1.81	144.000
4.0	79,200	26	23.7	1.79	142,000
4.4	79,700	35	31.6	1.81	144,000
6.7	79,200	14	12.8	1.83	145,000
7.1	79,200	18	16.2	1.80	143,000
8.8 12.65	79,700 79,500	30 30	27.4 27.7	1.83 1.85	145,000 147,000
11.0	79,200	31	28.6	1.84	146,000
15.1	79,700	55	51.0	1.85	148,000
20.0	79,700	45	42.3	1.88	149,000

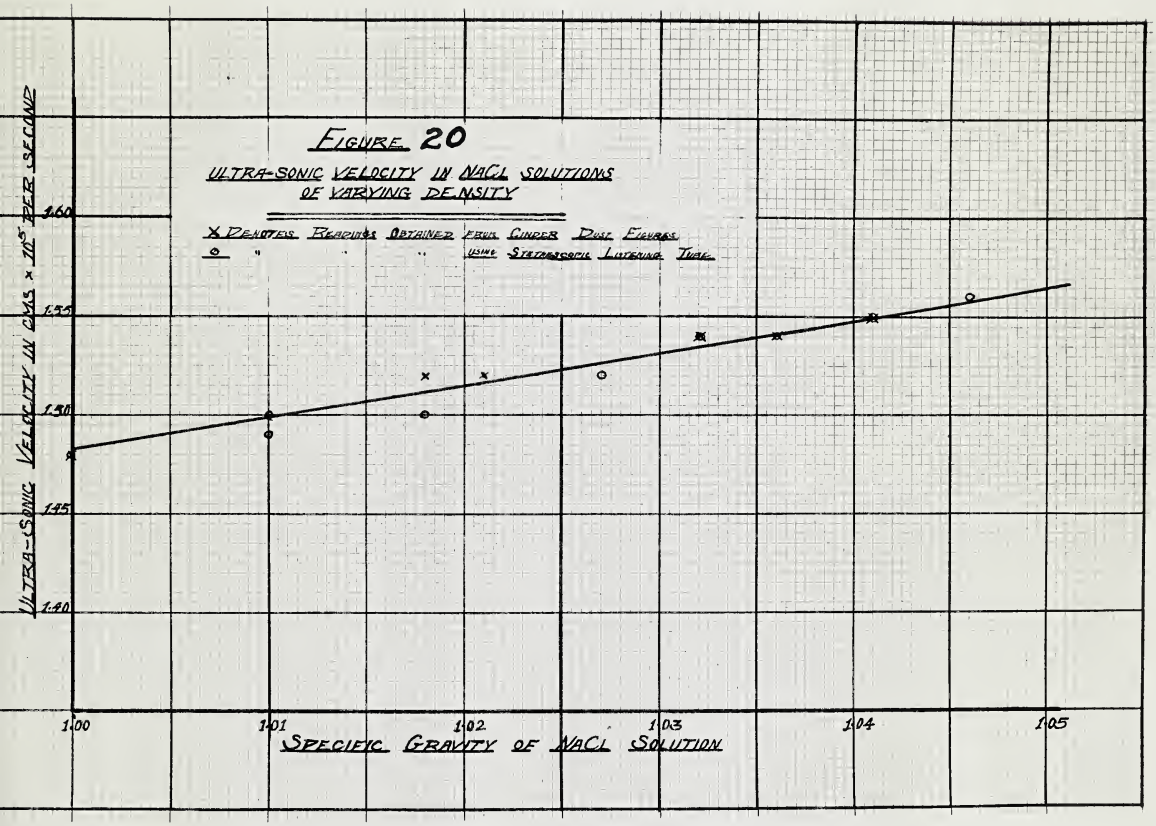


FIGURE 20
ULTRA-SONIC VELOCITY IN NaCl SOLUTIONS
OF VARYING DENSITY
X DENOTES READINGS OBTAINED FROM CINDER DUST FIGURES
O " " " USING STETHOSCOPIC LISTENING TUBE
ULTRA-SONIC VELOCITY IN CMS × 10^5 PER SECOND
1.60
1.35
1.30
1.25
1.20
1.00
1.01
1.02
1.03
1.04
1.05
SPECIFIC GRAVITY OF NaCl SOLUTION

Velocity in Salt Solution of Various Densities.

The change of the velocity in a salt solution with change of concentration was investigated. Table VIII gives the readings obtained. Figure 20 shows the velocity plotted as a function of the density of the solution. The average temperature was 17° C. There is a decided change in velocity with change in concentration and this may account for the change in velocity of the ultra-sonic waves in tap water used at different times of the year. At one time there may be a different quantity of salts in solution from that found at another time of the year.

TABLE VIII.

Listening Tube Method.

Specific Gravity.	n cycles/sec.	N	L cms.	Average λ cms.	V cms./sec.
1.000	80,200	34	31.4	1.84	147,800
1.010	80,200	30	28.0	1.86	148,800
1.010	80,200	30	28.1	1.88	150,100
1.018	80,200	30	28.4	1.88	150,300
1.027	80,400	30	28.5	1.90	152,100
1.032	80,400	40	38.1	1.91	153,700
1.036	80,400	28	27.0	1.93	154,200
1.042	80,400	40	38.7	1.93	155,200
1.046	80,600	50	48.8	1.94	156,000
		Dust Figure Method			
1.000	80,200	50	46.6	1.85	148,000
1.018	80,200	54	51.2	1.90	152,000
1.021	80,000	40	37.6	1.90	152,000
1.032	80,400	40	38.6	1.92	154,000
1.036	80,200	29	30.0	1.93	154,000
1.042	80,400	30	29.0	1.92	155,000

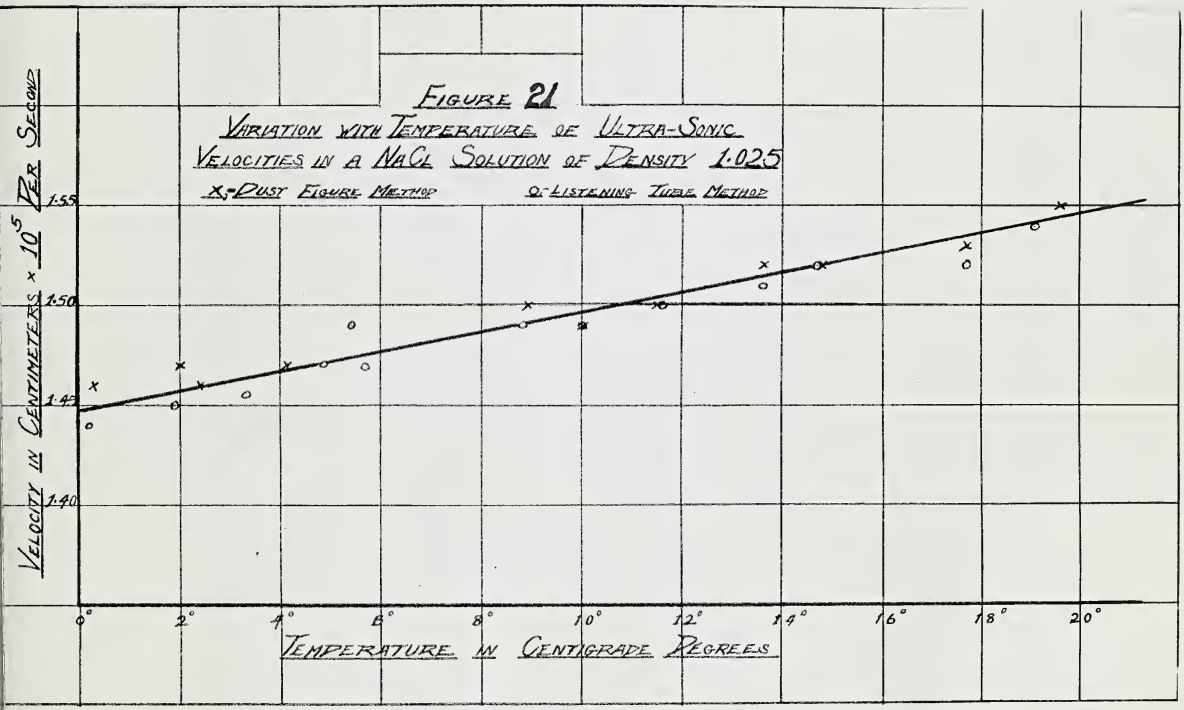


FIGURE 21
VARIATION WITH TEMPERATURE OF ULTRA-SONIC
VELOCITIES IN A NaCl SOLUTION OF DENSITY 1.025
X-DUST FIGURE METHOD
O-LISTENING TUBE METHOD
VELOCITY IN CENTIMETERS x 10^5 PER SECOND
1.55
1.50
1.45
1.40
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
20°
TEMPERATURE IN CENTIGRADE DEGREES

Velocity Change with Change of Temperature--
Salt Solution, Density 1.025 gms./c.c.

The change of velocity of ultra-sonic waves in a salt solution of constant density with change of temperature was studied and the results are tabulated in Table IX. Figure 21 shows graphically the relation between the two. The points forming the curve were arrived at in a manner similar to those in the graph showing change of velocity in water with change of temperature. See above.

TABLE IX.

Temperature °C.	n cycles/sec.	N	L cms.	Average λ cms.	V cms./sec.
Listening Tube Method.					
0.2	80,200	50	45.1	1.80	144,000
1.9	80,200	50	45.4	1.81	145,000
3.3	80,600	50	45.9	1.82	146,000
4.9	80,600	40	36.5	1.82	147,000
5.4	80,600	40	36.7	1.85	149,000
5.7	80,200	50	46.0	1.83	147,000
8.8	80,200	50	46.7	1.86	149,000
10.0	80,400	50	46.4	1.85	149,000
11.6	80,200	50	46.3	1.87	150,000
13.6	80,900	50	47.0	1.88	151,000
14.7	80,200	40	37.7	1.89	151,000
17.7	80,900	40	37.9	1.89	152,000
19.1	80,600	40	37.9	1.91	154,000
24.0	80,400	39	37.2	1.92	154,000
Dust Figure Method					
0.3	80,200	47	42.9	1.83	146,000
2.0	80,200	30	27.4	1.83	147,000
2.4	80,200	43	41.3	1.83	146,000
4.1	80,200	30	27.7	1.84	147,000
8.9	80,200	25	23.6	1.88	150,000
9.3	80,200	40	37.7	1.90	152,000
10.1	80,400	40	37.2	1.85	149,000
11.5	80,200	29	27.3	1.88	150,000
13.6	80,400	48	37.2	1.85	149,000
14.8	80,200	40	38.0	1.91	152,000
17.7	80,400	40	38.2	1.91	153,000
19.6	80,600	40	38.5	1.92	155,000
23.8	80,400	50	48.1	1.92	155,000

Velocity of Ultra-Sonic Waves in the following Liquids: Salt Solution, Ethyl Alcohol, Methyl Alcohol, Transformer Oil.

In Part I (c) the two methods for determining the velocity of ultra-sonic waves in liquids were described and the merits and demerits of each noted. On account of the difficulty there experienced in the listening method, the following experiments were carried out using the dust method exclusively and confining the liquid to a horizontal column about 30 cms. long. The small pile-of-plates transmitter was used as the source of ultra-sonic energy.

The following table gives the readings obtained with the calculated results:

TABLE X.

Voltage across Transmitter	Wave-Length cm.	Frequency of Oscillation ~/sec.	Velocity cm./sec.	Temperature °C.
Salt Solution--Density 1.025 gms./c.c.				
3700	1.66	95,700	158,900	15.4
3600	1.643	96,700	158,900	15.7
3300	1.388	113,000	156,900	15.8
Ethyl Alcohol--95% Solution.				
3100	1.118	113,000	126,300	16.4
3900	1.328	97,200	129,100	15.6
3200	1.565	85,200	133,300	15.2
Methyl Alcohol				
3600	1.257	99,300	124,800	17.3
3650	1.283	98,900	126,900	16.3
3650	1.057	118,700	125,500	15.5
3600	1.033	118,700	122,600	14.9
3300	0.835	142,400	118,900	14.0
Transformer Oil				
3800	1.75	85,000	148,900	18.7
4200	2.0	76,400	152,800	18.7
3300	1.39	112,600	156,300	19.2
4300	1.61	97,600	157,300	19.6

Stephenson, in the Physical Review, Volume 21, Second Series, No. 2., page 184, states his results in regard to the velocity of sound in seawater. He found that with a 3.35% salinity and temperature of -0.3° C., the average velocity of sound in seawater was equal to 1453.3 meters per second. According to curve shown in Figure 21, the velocity of the ultra-sonic energy at -0.3° C. is 1445.0 meters per second in a salt solution of 1.025 specific gravity.

It will be noticed in Table X that there is a decrease in the velocity of the ultra-sonic waves with an increase in frequency. This is particularly noticeable in the experiment with methyl alcohol, where a high frequency of 142,400 was chosen to study the effect.

In the case of transformer oil, it was noted during the experiment that with a high frequency (113,500 cycles per second) that the energy of the beam was dissipated and no stationary waves were formed with a 30 cm. column, but the temperature assumed a decided increase. However, with a 14 cm. column the ultra-sonic energy was great enough to form the required standing waves.

Also there was the suggested effect of the viscosity of the liquid diminishing the velocity as the frequency became higher.

Adiabatic Compressibility.

The formula $V = \sqrt{\frac{E}{d}}$, where V = velocity of the wave in a medium, E = the modulus of adiabatic compressibility of the medium, d = density, enables us to determine the adiabatic compressibility of the liquids used above, namely, water, salt solution, ethyl alcohol, methyl hydrate, and transformer oil. The density of each was determined by the specific gravity bottle method and the hydrometer method. The velocities used were the averages of those noted in Tables V and X.

In the following table are found the calculated results for bulk modulus, compressibility per bar (i.e., 10^6 dynes per sq. cm.) and compressibility per atmosphere for the above-mentioned liquids.

TABLE XI.

Liquid	Density gms./c.c.	Av. Vel. of u.s. waves in liquid cm./sec.	Av. Temperature °C.	Bulk Modulus x 10^{10}	Comp. C. per bar x 10^6	Comp. C. per atm. x 10^6
Water	1.000	150,000	15.0	2.25	44.4	44.9
Salt Solution	1.025	158,200	15.6	2.56	39.1	39.6
Ethyl Alcohol	0.896	129,600	15.7	1.354	73.8	74.7
Methyl Alcohol	0.819	124,900	16.0	1.278	78.3	79.2
Trans. Oil	0.879	153,700	19.0	2.076	48.1	48.7

Another observer Tyer' has determined the adiabatic compressibility of liquids by a static method in which he has subjected the liquids contained in a suitable vessel to a change in pressure of about one atmosphere, and then measured the volume change before any of the heat produced in compression was lost. In the case of water he found the adiabatic compressibility to be 46.81×10^{-6} at 14.12° C. and in the case of ethyl alcohol 93.80×10^{-6} at 15.52° C. (The compressibility is expressed per atmosphere.) He has no results for comparison with salt solution, methyl alcohol, or transformer oil, but there is fair agreement between his results and those given here in the case of water.

Conclusions.

Though the velocity of ultra-sonic energy is not greatly affected by the frequency of vibration, there is a slight suggestion of a diminishing velocity with increasing frequency. This would be due to the viscosity of the liquid.

The experimental error due to measurements was about 2%; the frequency was measured by a Hertzian wavemeter, which was subject to an error of 1.5%, giving a total experimental error of about 3.5%. If the velocity in water at ordinary temperature is chosen it is found that the average value over the range of frequencies used here is in the neighborhood of 150,000 cm./sec.

'Journal of Chemical Society, C III, Pt. 2, Transactions, 1913, p. 1682.

and the greatest deviation is not over 4000 cm./sec. This represents an error of nearly 3%, which is about the order of the experimental error.

The possible error in either method is about 3.5%. The curve shows that the velocity of ultra-sonic energy in pure water varies from 141,400 cm./sec. at 0° C. to 149,800 cm./sec. at 20°C., giving a variation of 5.9%. The variation in density of water over this range is 0.2%. The change in velocity must then be due chiefly to a change of the elasticity of the liquid.

In the temperature gradient experiment a large number of readings were taken in the neighborhood of 4° C. Water at this temperature has its maximum density and it was thought that there might be a change in the elastic properties which would be indicated by a change in the ultra-sonic velocity. However, no marked change was noted.

The addition of a small percentage of salt increases the velocity to a marked extent. This very likely accounts for the discrepancies in the velocities in tap water determined at various times of the year when there is a variation in the nature of the impurities in tap water.

The velocity determination by the ultra-sonic method is more accurate in the case of sea water than that determined under actual sea conditions, for in the ultra-sonic method the mass of water is of such dimensions that the temperature

is the same throughout, whereas such a condition cannot be obtained in the measurement at sea.

In regard to the adiabatic compressibilities, there are variations from the results of other observers; and this is noted in the case of Tyrer', who is the latest to do work of this kind. But a comparison of the two methods leads one to believe that the method here employed furnishes a means of getting more satisfactory results with a smaller chance for errors.

'Journal of Chemical Society, C III, Pt. 2, Transactions, 1913, p. 1682.

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